METHODS OF MEASURING INDUSTRY TOTAL FACTOR PRODUCTIVITY WITHIN AN INPUT-OUTPUT FRAMEWORK

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Abstract. This paper reconsiders the methods of measuring industry total factor productivity. The method offered by the honourable authors Ronald E. Miller and Peter D. Blair in their relevant book "Input-output analysis. Foundations and extensions. Second Edition. – Cambridge University Press, 2013" (14.2.1 Total Factor Productivity, pages 670-673) will be carefully tested. And it will be proved that the concept of the rate of total factor productivity growth offered in this book is inconsistent. With this, also decomposition of the change in total output of industry as "portion of change accounted for by using old technology to meet new input needs" plus "portion of change accounted for by using new technology to meet old input needs" is unsubstantial. The author realizes his serious liability for such categorical assertion about failure in the book presented under the famous "Cambridge University" brand and that is the reason why he bestows the authority upon the scientific conference to pass the sentence about correctness of his conclusions. In the second part of the present paper the original method of industry total factor productivity benchmarking within an input-output framework is offered. This method is grounded on the well-known Data Envelopment Analysis (DAE) using DAE modification by Jaunzems (2007). The main idea of this method is to compare the input-output capability of one and the same industry during one and the same time period in different countries. Thanks to the World Input-Output Database (WIOD) with its unified structured statistical information it is easy to carry out this method practically. Comparison of Latvia's agriculture industries A01, A02, A03 Input-Output converting relative efficiency with proper industries in the Baltic States and Finland (2000, 2014) gives original economic results never met before.

Key words: input-output model, total factor productivity, modified data envelope analysis, industries benchmarking.

Introduction

The object of productivity measurement is the unit of production, which produces multiple outputs using multiple inputs. The developments in economic productivity measuring within an input-output framework have been summarized in the Journal of the International Input-Output Association (Volume 19 Number 3, September 2007) [1]. In the book "Ronald E. Miller and Peter D. Blair. Input-Output Analysis. Foundations and Extensions. Second Edition. – Cambridge University Press, 2013" [2], the authors explore the concept of total factor productivity (TFP), which is defined generally as the growth in total output that is not attributable to growth in inputs (14.2.1 Total Factor Productivity, pages 670-673).

The current paper is devoted to two questions.

Firstly, the method offered by the honourable authors Ronald E. Miller and Peter D. Blair in their relevant book will be carefully tested. It will be proved that the concept of the rate of total factor productivity growth offered in this book is inconsistent. With this, also decomposition of the change in total output of industry as "portion of change accounted for by using old technology to meet new input needs" plus "portion of change accounted for by using new technology to meet old input needs" is unsubstantial. The author realizes his serious liability for such categorical assertion about failure in the book presented under the famous "Cambridge University" brand. However, at the same time, he asks to appreciate his conclusions if they are not wrong.

Secondly, the original method of industry total factor relative productivity measuring within an input-output framework grounded on the modification by Jaunzems (2007) of well-known Data Envelopment Analysis (DAE) is offered. The main idea of this method is to compare the input-output capability of definite industry with input-output capability of the proper industries in the reference group countries during one and the same fixed time period. This method can be applied also for research of input-output capability of one and the same industry from dynamic point of view in the definite country. Thanks to the World Input-Output Database (WIOD) [4] with its unified structured statistical information it is easy to carry out this method practically. In order to show this method in action the initial benchmarking of Latvian agriculture industries "Crop and animal production, hunting

DOI: 10.22616/ERDev2017.16.N077

and related service activities” (A01), “Forestry and logging” (A02), “Fishing and aquaculture” (A03) in set of the proper industries in the Baltic States and Finland is provided.

Materials and Methods

In the theoretical sense this research is based on application of linear algebra, linear programming and quantitative approaches to decision making as management science in economics and input-output analysis. In the more practical sense this article is based on the mentioned above respectable big scale (750 pages) book of Ronald E. Miller and Peter D. Blair [2] and on the famous and very widely used big scale (815 pages) book “David R. Anderson, Dennis J. Sweeney, Thomas A. Williams. An Introduction to Management Science: Quantitative Approaches to Decision Making. Seventh Edition.– West Publishing Company, 1994” [3]. The most important references in the current paper are to the two mentioned above books. The input-output model and the rate of total factor productivity growth is considered in connections with the book [2]. Economic Systems Research (2007) [1] contains articles summarizing developments in this area.

Let us shortly expound the theoretical framework and the methods used.

The input-output price model based on monetary data in current prices in scalar form can be expressed as follows:

(α0) *direct model*: $z_{i\,1} + z_{i\,2} + ... + z_{i\,n} + y_{i\,1} + y_{i\,2} + ... + y_{i\,m} = x_i$,

(β0) *dual model*: $z_{1\,j} + z_{2\,j} + ... + z_{n\,j} + (va)_j = x_j$, (i, j = 1, 2, .. . n).

Here $z_{i\,j}$ is the payment of j-th industry to the i-th industry for buying materials for intermediate use during production its total output x_j; (i, j = 1, 2, .. . n).

$y_{i\,k}$ is the k-th component of i-th industry final product (final demand), (i =, 2, ... , n;, k =1, 2, ... , m).

$(va)_j$ is the value added produced by j-th industry, (j = 1, 2, .. . n).

In order to be shorter we will use the matrix form of the input-output model:

(α1) *direct model* $Z\,\mathbf{1} + Y = X$,

(β1) *dual model* $Z^T\,\mathbf{1} + (VA) = X$,

where $X = (x_1\ x_2\ ...\ x_n)^T \in \mathbf{R}^{n,\,1}$ is total output;

$Y = (y_1\ y_2\ ...\ y_n)^T \in \in \mathbf{R}^{n,\,1}$ is the final product (final demand);

$y_i = y_{i\,1} + y_{i\,2} + ... + y_{i\,m}$, (i = 1, 2, .. . n);

$(VA) = ((va)_1\ (va)_2\ ...\ (va)_n)^T \in \mathbf{R}^{n,\,1}$ is the vector of total value added;

$Z = (z_{i\,j}) \in \mathbf{R}^{n,\,n}$ is the matrix of intermediate consumption in monetary terms.

Vector $\mathbf{1}= (1\ 1\ ...\ 1)^T \in \mathbf{R}^{n,\,1}$ is utilized for sum expounding in order to show components of the sum.

Under the axiom of linearity the input-output model is expressed in form

(α2) *direct model* $X = A\,X + Y$,

(β2) *dual model* $P = A^T\,P + V$,

where $A = (a_{i\,j}) \in \mathbf{R}^{n,\,n}$, (i, j = 1, 2, .. . n) is the matrix of technical coefficients (direct input coefficients), $a_{i\,j} = z_{i\,j} : x_j$, (i, j = 1, 2, .. . n);

$P = (p_1\ p_2\ ...\ p_n)^T \in \mathbf{R}^{n,\,1}$ is the vector of price indices,

$V = (v_1\ v_2\ ...\ v_n)^T \in \mathbf{R}^{n,\,1}$ is the vector of value added with respect to unit of total output,

$v_j = (va)_j : x_j$, (i, j = 1, 2, .. . n).

For exploring the input-output problem analysis for definite industry, definite year, definite country we are using the following notations:

(a) for rows: Z_{row}(industry; year; country), A_{row}(industry; year; country);

(b) for columns: Z_{col}(industry; year; country), A_{col}(industry; year; country).

The rate of total factor productivity growth in the book [2] is defined as

(γ) $$\tau_j = -\left(\sum_{i=1}^{n} d\,a_{i\,j} + d\,v_j\right).$$

In this paper the theoretical concept of total factor productivity (TFP) offered by Ronald E. Miller and Peter D. Blair is the object of detailed critical analysis. Also, the numerical example of total factor productivity provided by Ronald E. Miller and Peter D. Blair will be analyzed in detail.

The operating units very often have multiple inputs and multiple outputs. In such situations it is difficult for economists to determine, which operating units in the reference group are inefficient in converting their multiple inputs into multiple outputs in comparison with other unit capability. In that turn the Data Envelopment Analysis (DEA) is used, which is an application of linear programming to measure the relative efficiency of operating units with the same goals and objectives. The Data Envelopment Analysis as a method in operations research has a strong link to the production theory in economics. Practically the DEA is also used for benchmarking in operations management.

The main idea of the Data Envelopment Analysis is to compare the input-output capability of each separate reference group unit with theoretically constructed composite operating unit optimally based on all units in the reference group. Constraints in the linear programming model require all outputs of the composite unit to be greater than or equal to outputs of the unit being evaluated, but all inputs to be less than or equal to inputs of the unit being evaluated. In the present paper the Data Envelopment Analysis is examined following David R. Anderson, Dennis J. Sweeney, Thomas A. Williams [3]. Developments in DEA and the Mathematical Programming Approach to Frontier Analysis are analysed thanks to L. M. Seiford and R. M. Thrall (1990).

In the present paper the original method of industry total factor productivity measuring within an input-output framework grounded on the DAE modification by Jaunzems (2007) is offered.

The source of information for such measuring is the World Input-Output Database (WIOD) [5]. WIOD with its unified structured statistical information allows us to carry out this method easy practically. It is possible to construct the DAE model also on the bases of Supply tables and Use tables, which are compiled under unified standards [6]. A first version of the World Input-Output Database was constructed within the official WIOD Project, funded by the European Commission as part of the 7th Framework Programme, Theme 8: Socio-Economic Sciences and Humanities. Data for 56 sectors are classified according to the International Standard Industrial Classification revision 4. The National Input-Output Tables are compiled in current prices, expressed in millions of US dollars.

Results and Discussion

The results and discussion are stated in two short chapters.

In the first chapter the author examines in details the rate of total factor productivity growth offered by Ronald E. Miller and Peter D. Blair and proves that this concept is inconsistent. In the second chapter, the original method of measuring industry total factor relative productivity within an input-output framework is examined. As an numerical example, containing original economic results, the comparison of agriculture industries in the Baltic States and Finland (2000, 2014) is provided.

1. The concept of rate of total factor productivity growth offered by Ronald E. Miller and Peter D. Blair. The following text, except analysis and criticism, is partially cited from the book [2], pages 670-673.

1.1. Theoretical part

The authors begin with the fundamental input-output accounting relationship

$$x_j = \sum_{j=1}^{n} a_{ij} x_j + v_j x_j = \left(\sum_{i=1}^{n} a_{ij} + v_j \right) x_j . \tag{1}$$

Using the rule for differential of product the authors get equation

$$dx_j = d\left[\left(\sum_{i=1}^{n} a_{ij} + v_j \right) x_j \right] = \left(\sum_{i=1}^{n} a_{ij} + v_j \right) dx_j + \left(\sum_{i=1}^{n} d\,a_{ij} + d\,v_j \right) x_j . \tag{2}$$

The authors asserts that the rate of total factor productivity growth often is defined as

$$\tau_j = -\left(\sum_{i=1}^{n} d\,a_{ij} + d\,v_j \right) . \tag{3}$$

And so, equation (2) becomes

$$\mathrm{d}\,x_j = \left(\sum_{i=1}^{n} a_{ij} + v_j\right) \mathrm{d}\,x_j - \tau_j\, x_j\,. \tag{4}$$

In order to make use of available input-output data it is usual to express the relationships in (2) and (3) in finite-difference form, where $\mathrm{d}\,x_j \approx \Delta\, x_j = x_j^1 - x_j^0$, $\mathrm{d}\,a_{ij} \approx \Delta\, a_{ij} = a_{ij}^1 - a_{ij}^0$, $\mathrm{d}\,v_j \approx \Delta\, v_j = v_j^1 - v_j^0$. Ignoring "second order" effects, (2) becomes

$$x_j^1 - x_j^0 = \Delta\, x_j = \left(\sum_{i=1}^{n} a_{ij}^0 + v_j^0\right) x_j^1 - \left(\sum_{i=1}^{n} a_{ij}^0 + v_j^0\right) x_j^0 + \tag{5}$$

$$+ \left(\sum_{i=1}^{n} a_{ij}^1 + v_j^1\right) x_j^0 - \left(\sum_{i=1}^{n} a_{ij}^0 + v_j^0\right) x_j^0\,.$$

The authors of the book [2] interpret the equation (5) as decomposition:

"portion of change accounted for by using old technology, as reflected in a_{ij}^0 and v_j^0, to meet new input needs" plus "portion of change accounted for by using new technology, as reflected in a_{ij}^1 and v_j^1, to meet old input needs".

In finite-difference form (3) is

$$\tau_j = -\left(\sum_{i=1}^{n} \Delta\, a_{ij} + \Delta\, v_j\right) \tag{6}$$

so

$$\Delta\, x_j = \left(\sum_{i=1}^{n} a_{ij} + v_j\right) \Delta\, x_j - \tau_j\, x_j^0\,. \tag{7}$$

Now follows the author's criticism of the concept of the rate of total factor productivity growth. First of all, let us notice that input-output systems are balanced and therefore

$$\sum_{j=1}^{n} a_{ij} + v_j \equiv 1. \tag{8}$$

Therefore, the equation (1) simply means that $x_j \equiv x_j$.

Taking in account (8), we get $\mathrm{d}\left(\sum_{i=1}^{n} a_{ij} + v_j\right) = \left(\sum_{i=1}^{n} \mathrm{d}\,a_{ij} + \mathrm{d}\,v_j\right) \equiv 0.$

It is obvious that equation (2) simply means that $\mathrm{d}\,x_j = \mathrm{d}\,x_j$.

The rate of total factor productivity growth τ_j is inconsistent, because it always equals zero:

$$\tau_j = -\left(\sum_{i=1}^{n} \mathrm{d}\,a_{ij} + \mathrm{d}\,v_j\right) \equiv 0. \tag{9}$$

The interpretation of the equation (5) as decomposition does not give a consistent result, because

$$\left(\sum_{i=1}^{n} a_{ij}^0 + v_j^0\right) \equiv 1,\ \left(\sum_{i=1}^{n} a_{ij}^1 + v_j^1\right) \equiv 1.$$

1.2. Numerical example: Total Factor Productivity.

In order to be more understandable the honourable authors Ronald E. Miller and Peter D. Blair provide a numerical example [2; pages 672, 673] . Let us verify their calculations.

The authors offer an input-output economy, for which technical coefficients and value added in three successive years are defined by

$$\mathbf{A}^{(0)} = \begin{bmatrix} .233 & .323 & .326 \\ .116 & .242 & .130 \\ .186 & .274 & .380 \end{bmatrix} \text{ and } \mathbf{v}^{(0)} = \begin{bmatrix} .465 \\ .161 \\ .163 \end{bmatrix} \text{ for year 0;}$$

$$\mathbf{A}^{(1)} = \begin{bmatrix} .120 & .244 & .246 \\ .060 & .183 & .098 \\ .096 & .207 & .287 \end{bmatrix} \text{ and } \mathbf{v}^{(1)} = \begin{bmatrix} .723 \\ .366 \\ .369 \end{bmatrix} \text{ for year 1;}$$

$$\mathbf{A}^{(2)} = \begin{bmatrix} .078 & .108 & .109 \\ .039 & .081 & .043 \\ .062 & .091 & .127 \end{bmatrix} \text{ and } \mathbf{v}^{(2)} = \begin{bmatrix} .465 \\ .161 \\ .163 \end{bmatrix} \text{ for year 2.}$$

Ronald E. Miller and Peter D. Blair calculate that for this example the total factor productivity growth vector is $\boldsymbol{\tau}^{(1\,0)} = (0\ 0\ 0)^T$, but $\boldsymbol{\tau}^{(2\,1)} = (0.357\ 0.559\ 0.558)^T$.

Let us remember that in finite-difference form

$$\tau_j^{(1\,0)} = -\left(\sum_{i=1}^{n} \Delta a_{ij} + \Delta v_j\right) = \left(\sum_{i=1}^{n} a_{ij}^0 + v_j^0\right) - \left(\sum_{i=1}^{n} a_{ij}^1 + v_j^1\right). \tag{10}$$

We already showed above that

$$\left(\sum_{i=1}^{n} a_{ij}^0 + v_j^0\right) \equiv 1, \left(\sum_{i=1}^{n} a_{ij}^1 + v_j^1\right) \equiv 1$$

and now we may test these identities numerically. It is easy to check that $\tau_j^{(1\,0)} \equiv 0$ for $\forall\, j \in \{1, 2, 3\}$.

Analogically

$$\tau_j^{(2\,1)} = \left(\sum_{i=1}^{n} a_{ij}^1 + v_j^1\right) - \left(\sum_{i=1}^{n} a_{ij}^2 + v_j^2\right) \equiv 0 \text{ for } \forall\, j \in \{1, 2, 3\}. \tag{11}$$

But now the reason arises: “Why in the numerical example given by Ronald E. Miller and Peter D. Blair the total factor productivity vector is $\boldsymbol{\tau}^{(2\,1)} = (0.357\ 0.559\ 0.558)^T$?”

How we can explain this contradiction?

The explanation is very simple. There is a primitive mistake in the calculations by Ronald E. Miller and Peter D. Blair: for year 2 the same vector of value added is taken as for year 0. In this case, naturally, equation

$$\left(\sum_{i=1}^{n} a_{ij}^2 + v_j^2\right) \equiv 1$$

does not fulfil. The correct vector of value added for year 2 can be calculated as follows:

$$\mathbf{v}^{(2)} = \begin{bmatrix} 1-(0.078+0.039+0.062) \\ 1-(0.108+0.081+0.091) \\ 1-(0.109+0.043+0.127) \end{bmatrix} = \begin{bmatrix} .821 \\ .718 \\ .721 \end{bmatrix}.$$

2. *The method* of industry total factor relative productivity measuring within an input-output framework.

2.1. *Construction of theoretical* composite industry as benchmark based of all proper industries in the reference group.

The industries have multiple inputs and multiple outputs. We are going to compare the efficiency of converting their multiple inputs into multiple outputs of the one and the same industry in fixed time period in different countries.

How in such a situation is it possible to determine which industries in the reference group are inefficient in converting their multiple inputs into multiple outputs?

First of all, we need unified structured input-output tables for the reference group.

Let us assume that we have unified structured information in form (α1), (β1) for each country of the reference group.

Let us assume that we are interested in relative effectiveness of j-th industry in year t in the c-th country of the reference group. With help of linear programming we will compare the input-output capability of this chosen i-th industry with theoretically constructed composite j-th industry optimally based of all j-th industries in the reference group. The form of benchmark is determined by the most efficient industries. Constraints in the linear programming model require all outputs of the composite unit to be greater than or equal to outputs of the unit being evaluated, but all inputs to be less than or equal to inputs of the unit being evaluated.

In general, the constraints and the objective can be written as follows. (The author hopes that explaining explanations for the notations used above allows to understand these unexplained notations.)

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with respect to the constraints:

w_1 INPUT(industry_j; year_t; country_1) + w_2 INPUT(industry_j; year_t; country_2) +... + w_r INPUT(industry_j; year_t; country_r) $\leq$ s $\cdot$ INPUT(industry_j; year_t; country_r_0)

w_1 OUTPUT(industry_j; year_t; country_1) + w_2 OUTPUT (industry_j; year_t; country_2) +... + w_r OUTPUT (industry_j; year_t; country_r) $\geq$ OUTPUT(industry_j; year_t; country_ r_0)

$r_0 \in \{1, 2, \ldots, r\}$

$w_1 \geq 0, w_2 \geq 0, \ldots, w_r \geq 0.$

The effectiveness of the converting multiple inputs into multiple outputs for definite j-th industry in the country r_0 can be evaluated by comparing the capability of that industry with the capability of composite j-th industry optimally constructed based on more suitable attributes of all j-th industries of the reference group. The most efficient industries form a "composite industry". The interpretation of "composite industry" is rather abstract. The numbers $w_1^*, w_2^*, \ldots, w_r^*$ show the latent reserve of (industry_j; year_t; country_r_0) efficiency. The "composite industry" must be used as benchmark in operations management, it shows the ruling attributes of which industries must be transferred and implemented in the j-th industry of country r_0.

Of course, these constraints can be modified, also the optimality can be defined in different ways, as it is done in the mentioned above paper of Jaunzems (2007).

2.2. *Numerical example*: comparison of Latvia's agriculture industries A01, A02, A03 with proper industries in the Baltic States and Finland (2000, 2014).

The wide research of Latvia's economy benchmarking made by the author will be published in the nearest future. Now, in order to demonstrate the offered method in action, let us compare the input-output converting capability of Latvian agriculture industries "Crop and animal production, hunting and related service activities" (A01), "Forestry and logging" (A02), "Fishing and aquaculture" (A03) with the composite industries constructed with help of proper industries in the Baltic States and Finland. The World Input-Output Database provides us unified structured input-output tables for such research. As linear programming software the Microsoft Excel procedure Solver is applied.

In practical applications of the industry total factor relative productivity measuring method within an input-output framework the problem arises of how to define input and output of industry. There can be different input and output selection possibilities depending on the goals of the research.

In this paper we consider the vector (total intermediate domestic consumption; total intermediate import consumption) as input of industry, but as output – value added of industry. We keep the value added constant and calculate Pareto efficient total intermediate consumption (domestic and import).

Conformable to the WIOD standards value added and intermediate consumption are provided in current (nominal) prices, expressed in millions of US dollars. In order to be shorter we will speak about the units of value.

2.2.1. *Let us examine* at first only the Baltic States (LTA; LTU; EST). We compare with respect to Pareto efficiency Latvia's industries A01, A02, A03 with optimally constructed composite proper industry. The results of calculation are shown in Table 1. The economic indicators are compiled in current prices, expressed in millions of US dollars.

Table 1

Relative effectiveness of Latvia's industries A01, A02, A03 in comparison with optimal proper composite industry of (LTA; LTU; EST) in 2000, 2014

Industry, year	Value added	Total intermediate domestic consumption	Total intermediate import consumption	Composite industry	Value added	Total intermediate domestic consumption	Total intermediate import consumption
A01, 2000	227	187	86	(0; 0; 0.39)	227	173 = 0.92·187	52 = 0.60·86
A01, 2014	451	701	565	(0; 0; 0.99)	451	444 = 0.63·701	250 = 0.43·565
A02, 2000	110	106	21	(0; 1.83; 0)	110	57 = 0.54·106	18 = 0.87·21
A02, 2014	424	638	163	(0; 2.02; 0)	424	333 = 0.52·638	160 = 0.98·163
A03, 2000	27	27	14	(0; 3.86; 0)	27	15 = 0.57·27	8 = 0.55·14
A03, 2014	32	23	22	(0; 0; 0.59)	32	15 = 0.67·23	15 = 0.67·22

Let us interpret the meaning of Table 1 with help of, for example, the rows A01, 2000 and A01, 2014.

Industry A01, 2000. Latvia's industry A01 in 2000 produced 227 value added units by total intermediate consumption of 273 units. These 273 units consist of 187 domestic units plus 86 import units. Optimal composite industry based on the reference group (LTA; LTU; EST) is (0; 0.39; 0). A "composite producer" has been formed from the most efficient producer – Estonia's A01 industry. That means, if the technology and management processes of industry A01 in Latvia would be organized with the same efficiency as Estonia's A01 industry, taking 39 % of it, than the same value added (227 units) can be produced by spending only 173 domestic units plus 52 import units. Therefore, for composite industry A01 intermediate domestic consumption is only 92 % from actual domestic consumption and import consumption is only 60 % of actual import consumption. These numbers show the latent reserve of industry A01 efficiency increasing in Latvia during investigating and transferring the appropriate ruling attributes from Estonia's A01 industry. This result has to be used for benchmarking in operations management.

Industry A01, 2014. Latvian industry A01 in 2014 produced 451 millions of dollars value added by total intermediate consumption of 1266 millions of dollars. If the technology and management processes of industry A01 in Latvia would be organized with the same efficiency as Estonia's A01 industry, than the same value added (451 millions of dollars) could be produced by spending only 444 millions of dollars as total intermediate domestic consumption (63 % of Latvia's actual total intermediate domestic consumption in industry A01) plus by spending 250 millions of dollars as total intermediate import consumption (43 % of Latvia's actual total intermediate import consumption in industry A01). Optimal composite industry is denoted as (0; 0; 0.99). Like in 2000, a "composite industry" has been formed from the most efficient producer – Estonia's A01 industry.

The interpretations of other rows of Table 1 are similar.

The following three figures depict the content of Table 1 graphically.

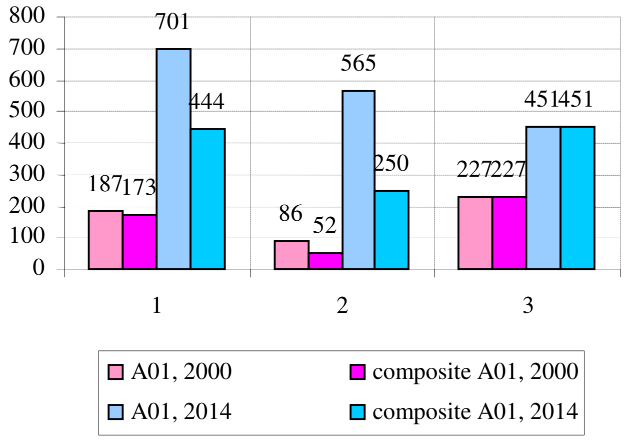

Fig. 1. **Latvia, A01:** “1”– A01 actual and composite intermediate domestic consumption; “2”– A01 actual and composite intermediate import consumption; “3”– A01 actual and composite value added

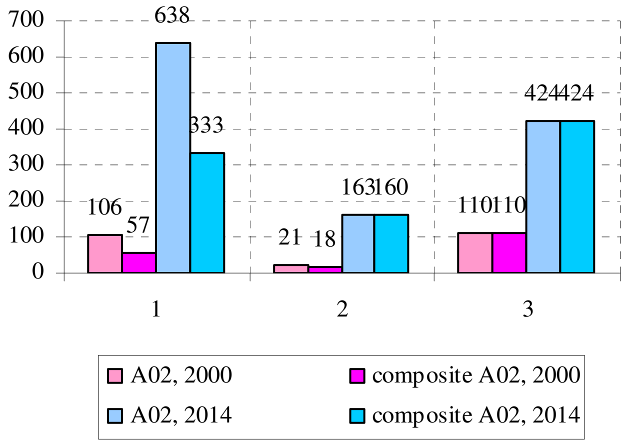

Fig. 2. **Latvia, A02:** “1”– A02 actual and composite intermediate domestic consumption; “2”– A02 actual and composite intermediate import consumption; “3”– A02 actual and composite value added

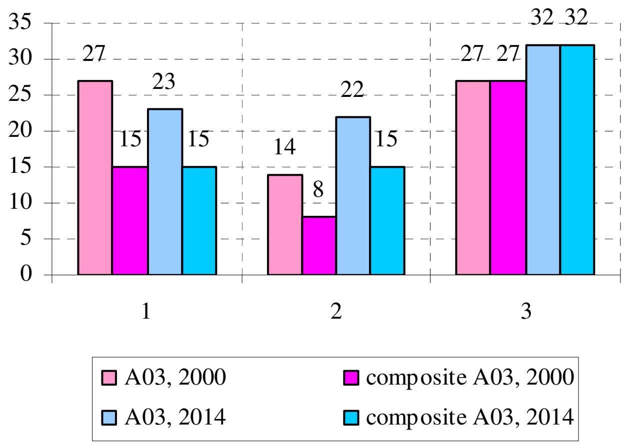

Fig. 3. **Latvia, A03:** “1”– A03 actual and composite intermediate domestic consumption; “2”– A03 actual and composite intermediate import consumption; “3”– A03 actual and composite value added

2.2.2. *Now we compare* with respect to Pareto efficiency Latvia’s industries A01, A02, A03 with artificially constructed optimal composite industry utilizing proper industries in the Baltic States and Finland (LTA; LTU; EST; FIN). The results of calculation are shown in Table 2.

The National Input-Output Tables are compiled in current prices, expressed in millions of US dollars.

Table 2

Relative effectiveness of Latvia's industries A01, A02, A03 in comparison with optimal proper composite industry of (LTA; LTU; EST; FIN) in 2000, 2014

Industry, year	Value added	Total intermediate domestic consumption	Total intermediate import consumption	Composite industry	Value added	Total intermediate domestic consumption	Total intermediate import consumption
A01, 2000	227	187	86	(0; 0; 0.39; 0)	227	173 = 0.92·187	52 = 0.60·86
A01, 2014	451	701	565	(0; 0; 0.99; 0)	451	444 = 0.63·701	250 = 0.43·565
A02, 2000	110	106	21	(0; 0; 0; 0.05)	110	29 = 0.28·106	5 = 0.25·21
A02, 2014	424	638	163	(0; 0; 0; 0.10)	424	140 = 0.22·638	25 = 0.15·163
A03, 2000	27	27	14	(0; 0; 0; 0.35)	27	15 = 0.56·27	2 = 0.15·14
A03, 2014	32	23	22	(0; 0; 0; 0.21)	32	9 = 0.38·23	9 = 0.41·22

The interpretations of other rows of Table 2 are similar to the interpretation of the rows of Table 1. The only difference is that now we take the Baltic States and Finland as the reference group.

Calculations show that for industry A01 "composite producer" has been formed from the Estonia's A01 industry. The "composite producer" in industry A02 and industry A03 has to be constructed by utilizing Finland's proper industry.

The figures depicting the content Table 2 graphically are omitted due to limited volume of the paper.

Conclusions

1. It is proved in this paper that the concept of the rate of total factor productivity growth, what often is defined as $\tau_j = -\left(\sum_{i=1}^{n} d\,a_{ij} + d\,v_j\right)$, is inconsistent. Also, decomposition of the change in total output of industry as "portion of change accounted for by using old technology to meet new input needs" plus "portion of change accounted for by using new technology to meet old input needs" offered in the book [2; page 671] is unsubstantial.
2. Practical applications allow to make a conclusion that the Modified Data Enveloped Analysis (MDEA) offered by Jaunzems (2006) is a suitable tool for measuring the relative efficiency of industry input-output capability comparing with the theoretically constructed composite industry optimally based on all proper industries in the reference group. Practical applications are possible thanks to the World Input-Output Database (WIOD) unified structured input-output tables (IOT) and thanks to linear programming (LP) software.
3. Comparison of Latvia's agriculture industries A01, A02, A03 Input-Output converting capability with proper industries in the Baltic States (2000, 2014) gives original economic results never met before. It turns out that technologies and operations management in these industries in Latvia are relatively low. The composite industries as benchmarks signalize: Estonia's industries A01 and A03 have to be used as benchmark for Latvia's A01 and A03. Lithuania's industry A02 has to be used as benchmark for Latvia's A02. The investigation will be continued.
4. Comparison of Latvia's agriculture industries A01, A02, A03 input-output converting relative efficiency with proper industries in the Baltic States and Finland (2000, 2014) gives original economic results never met before. It comes out that technologies and operations management in these industries in Latvia are relatively low. The composite industries as benchmarks signalize: Estonia's industry A01 has to be used as benchmark for Latvia's A01, but Finland's A02, A03 as benchmarks for Latvia's A02, A03. The investigation will be continued.

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